

Graph Theory

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ISBN 1466254998

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Preface

Graph theory, as a branch of mathematics, has a glorious history: from Euler's seven bridges of Königsberg in 1756, to the elusive proof of the four colour theorem in 2000, and beyond. Graph theory is of practical importance to computer scientists for designing efficient algorithms, because many of the hardest (NP-complete) problems are essentially graph theoretic in nature. In recent times, graph theory has found diverse and often unexpected applications in science, engineering and technology. Seemingly difficult problems become easy to solve when expressed in the proper graph theoretical context. The powerful combinatorial methods of graph theory are being used to discover and prove significant new results in a variety of areas of pure mathematics itself. Indeed, the past few decades have witnessed a rising level of interest and growing activity among mathematicians, scientists and engineers in graph theory. As a result, one finds graph theory as a vital component of the mathematics curriculum in colleges and universities all over the world. In India, the model syllabus for graduate level mathematics proposed by the University Grants Commission includes graph theory as a recommended course.

This book has grown from our experience over the past several years in teaching various topics in graph theory, at both the graduate and undergraduate levels. As the number of students opting for graph theory is rapidly increasing, an attempt has been made to provide the latest and best available information on the subject. Our aim is to present the basics of graph theory in such a way that an average student can acquire as much depth and comprehension as possible in a first course.

The book is primarily intended for use as a textbook at the graduate level (for students pursuing masters in mathematics and computer science), with the first eleven chapters forming a one year course. However, the first eight chapters may be used as a one semester course at the undergraduate level for students of computer science and engineering. The final sections of many chapters introduce advanced topics and unsolved problems that are the object of current research in

graph theory. Thus, the book can also be used by students pursuing research work in M. Phil and Ph. D. programmes.

There are many new topics in this book that have not appeared before in print: new proofs of various classical theorems, signed degree sequences, criteria for graphical sequences, eccentric sequences, matching and decomposition of planar graphs into trees. Scores in digraphs appear for the first time in print and the climax of the book is a new proof of the famous four colour theorem.

Many earlier books, monographs and articles have been used in the preparation of this book and we have included a comprehensive bibliography at the end of the book.

We would like to thank the Canadian Mathematical Society and the Math Forum at Drexel University for announcing the new proof of the four colour theorem in 2000. We are extremely grateful to the University of Kashmir and the Institute of Mathematics (Gurgaon) for their support during the writing of this book. Our sincere thanks go to Merajuddin (AMU, Aligarh), M. A. Sofi (University of Kashmir), Petrovic Vojislav (Novi Sad University), Ivanyi Antal (Eotvos Lorand University, Hungary), Zhou Guofei (Nanjing University, China), M. R. Sridharan (IIT, Kanpur), V. Krishnamurthy (BITS, Pilani), Niels Karlsson (Akureyri University), John-Tagore Tevet and Jüri Martin (Euroniversity, Tallinn), Anita Pasotti (Universita degli Studi di Brescia) and Vladimir Khachatryan (SUNY, Stony Brook) for their valuable suggestions and encouragement. We thank all our friends and colleagues, especially T. A. Chishti (University of Kashmir) and all the members of the Institute of Mathematics (Gurgaon) for supporting our work. We thank our research students, especially T.A. Naikoo for help in drawing the figures and carefully proof reading the manuscript. We are grateful to our families for their love and support during the time this book was being written. Finally, it is a pleasure to thank the management and staff of Orient Longman and Universities Press (India) for their interest, cooperation, and fine workmanship.

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