The Cosmological Constant

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Abstract

We show how to calculate Einstein's cosmological constant using the Grand Unified Theory. Using the topological properties of the gauge, we calculate the exact percentages of ordinary baryonic matter, dark matter and dark energy in the universe. These values are in perfect agreement with the seven-year Wilkinson Microwave Anisotropy Probe (WMAP) observations. Thus dark matter, dark energy and the cosmological constant are intrinsic properties of the gauge in the Grand Unified Theory.

In the grand unification of the standard model of particle physics with quantum gravity [1], Einstein's law of gravity for empty space is written in its simplest form as

$$R_{\mu\nu} = 0 \tag{1}$$

where $R_{\mu\nu}$ is the Ricci tensor [1, §7.4.1]. Einstein [2] considered a generalization of equation (1) by introducing the *cosmological constant* Λ to obtain

$$R_{\mu\nu} = \Lambda g_{\mu\nu} \tag{2}$$

where the $g_{\mu\nu}$ are given as functions of the space-time coordinates and define the space-time metric [1, §7.4.1]. We follow the Lambda-Cold Dark Matter (ΛCDM) model [1, §8], aka the standard model of big bang cosmology. In the ΛCDM model, the most commonly used solution $g_{\mu\nu}$ of equation (2) is known as the Friedman-Lemaître metric [3][4]. Since equation (1) is already in good agreement with observation for the solar system (almost flat space-time), Λ must be small enough not to disturb this agreement. Since the Ricci tensor $R_{\mu\nu}$ contains second derivatives of the metric tensor $g_{\mu\nu}$, the cosmological constant Λ must have dimensions $(\text{distance})^{-2}$. For Λ to be small this distance must be very large. It is a cosmological distance, of the order of the radius of the universe.

The cosmological constant Λ is equivalent to an intrinsic energy density of the vacuum. A positive vacuum energy density Λ creates a negative pressure which causes an accelerated expansion of the universe. Observational evidence that this is indeed the case was found by astronomers studying supernovae in 1998 [5]. The positive vacuum energy Λ is called *dark energy*. Calculating the exact percentage of dark energy in the total mass-energy of the universe is equivalent to calculating the value of the cosmological constant Λ. In 1934, while measuring the orbital velocities of galaxies in clusters astronomers found a discrepancy between the observed mass and the mass calculated from the equations of motion of the galaxies [6]. The galaxies must contain *dark matter* which cannot be observed i.e. a form of matter which does not emmit any electromagnetic radiation. Thus, the total energy-mass of the universe in the ΛCDM model arises from ordinary matter, dark matter and dark energy. The percentages of ordinary matter, dark matter and dark energy in the universe have been experimentally measured with great accuracy by the Wilkinson Microwave Anisotropy Probe (WMAP) [7] in 2010. We shall show how to calculate the percentages of ordinary matter, dark matter and dark energy from the grand unified theory [1] and corroborate the WMAP data.

In the grand unified theory [1], the particle frame defines the gauge. Thus, at each point of space-time there is an associated particle frame. In [1, §7.4.2], we have shown how the particle frame is embedded without self-intersection at each point of space-time. Each Schrödinger disc of a particle frame carries the curvature and Ricci tensors according to equation (1). We shall show that the particle frame also carries the cosmological constant Λ according to equation (2) and calculate the exact value of the cosmological constant Λ from the topological structure of the gauge.

Fix a Planck time interval T during the present epoch in the cosmological timeline [1, §4, §8]. During this Planck time interval T we may observe all the fermions in the universe which constitute the totality of ordinary matter. First consider a particular fermion *F*. The fermion *F* is selected according to the Fermion Selection Rule [1, §4.1] from its particle frame. The spin 1/2 fermion *F* satisfies Dirac's relativistic wave equation [8, §3] and determines exactly one of the four components Ψ_i (i = 0, 1, 2 or 3) of the wave function $\mathbf{\Psi}$. Thus, *F* determines exactly one half-surface out of the four half-surfaces that constitute the labeled *t*-Riemann surface of the particle frame, cf. [1, §4.4 The Spin Rule]. This half-surface consists of 6 Schrödinger discs, one of which is selected for the fermion *F*.

Since the 24 Schrödinger discs of the particle frame are superposed at a space-time point [1, §7.4.2], the non-zero amplitude of the Schrödinger wave function of *F* is replicated along the boundaries of all 24 Schrödinger discs of the particle frame. However, exactly one of the discs actually carries an electric charge $\underline{0}$ = 0, $\underline{1}$ =1/3, $\underline{2}$ = 2/3 or $\underline{3}$ = 1 (this is the Schrödinger disc of *F*). The sign of the electric charge is given by the half-surface determined by *F* [1, §4.5 The Electric Charge Rule]. The remaining 23 Schrödinger discs carry a replicated charge $\underline{0}$, $\underline{1}$, $\underline{2}$ or $\underline{3}$, but this charge is now interpreted as a gravitational gauge charge. This explains the reappearance of $\underline{0}$, $\underline{1}$, $\underline{2}$, $\underline{3}$ along with σ as the five parameters of the gravitational gauge group **SU(5)** in the grand unified theory [1, §7.4.6].

We postulate that the 5 remaining Schrödinger discs of the half-surface determined by *F* behave as dark matter. Since each of the 5 Schrödinger discs of dark matter have the same Schrödinger wave function as *F*, they will exhibit the same mass-energy as 5 copies of *F*. However, since these 5 Schrödinger discs do not have any electric charge defined, they cannot be observed via electromagnetic interactions i.e. will be dark matter. Nevertheless, the 5 Schrödinger discs of dark matter do have a gravitational gauge charge that can be observed over large distances via gravitational interactions.

We further postulate that the remaining 18 Schrödinger discs of the other 3 half-surfaces of the particle frame of *F* behave as dark energy. By the same argument, their effect can only be observed through gravitational interactions over large distances. Thus, the spin 2 graviton [1, §6.6.1, §6.6.2] must behave in two distinct ways. For the Schrödinger disc of the fermion *F* and the 5 Schrödinger discs of dark matter on the particle frame, it behaves as an attractive force carrier during gravitational interactions. Whereas, for the 18 Schrödinger discs of dark energy, it behaves as a repulsive force carrier during gravitational interactions.

For all space-time points where there is vacuum, we have a particle frame with no Schrödinger disc selected [1, §4 Figure 4.1]. By the above argument, we shall regard all 24 Schrödinger discs of such a particle frame to represent dark energy.

Each fermion F has a positive mass attributed to it by a Higgs particle H via the Higgs-Kibble mechanism [1][9]. We can pair all fermions with their associated Higgs particles (F, H) during the Planck time interval T. Thus, on the particle frame of F we can superpose the particle frame of H [1, §6.7.1, §6.7.2]. The Higgs particle H consists of 12 Schrödinger discs by the Higgs selection rule [1][9] and only one of the 12 Schrödinger discs will intersect with the Schrödinger disc of F, contributing 1/12 of the mass of F. Now consider all such Fermion-Higgs pairs in the universe during this Planck time interval T. The total observable mass must be in the ratio (1 + 1/12)/24, since each particle frame has 24 Schrödinger discs.

Thus, the percentage of ordinary observable matter in the universe is

$$M = 100 \times (1 + 1/12)/24 = 4.51\% \qquad (3)$$

Similarly, by the above argument, the percentage of dark matter in the universe is

$$D.M. = 100 \times 5 \times (1 + 1/12)/24 = 22.57\% \qquad (4)$$

Let us assume that the other massive bosons do not contribute much to the total mass-energy of the universe during the Planck time interval T. By the law of conservation of the total mass-energy of the universe, the remaining Schrödinger discs of all particle frames must make up the deficit in terms of dark energy. Hence, the percentage of dark energy in the universe is

$$D.E. = 100 - 4.51 - 22.57 = 72.92\% \qquad (5)$$

In particular, we have calculated the exact value of the cosmological constant Λ from the grand unified theory [1]. We have shown that ordinary matter, dark matter and dark energy are all properties of the particle frame which forms the gauge for the grand unified theory [1]. The percetages (3), (4) and (5) are in perfect agreement with the Seven-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations [7]:

$M = 4.5\% - 4.7\%$; $D.M. = 22\% - 24.6\%$; $D.E. = 70.6\% - 73.6\%$.

References

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